Engineering Structures 31 (2009) 2735–2743

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Multidomain finite and discrete elements method for impact analysis of a concrete structure

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ARTICLE INFO

Article history:
Received 27 February 2009
Received in revised form
17 June 2009
Accepted 2 July 2009
Available online 17 July 2009

Keywords:
Discrete elements method
Fast transient dynamics
Concrete
Impact
Spurious waves

ABSTRACT

This article focuses on a formulation for coupling discrete and finite element methods. The efficiency of the discrete element method for studying the fracture of heterogeneous media has been demonstrated, but it is limited by the number of elements. A multidomain analysis is thus proposed in order to reduce the computational effort. The structure is split into two subdomains, in each of which the method is adapted to the behavior of the structure under impact. The DEM is used to model the media close to the impacts. It easily takes into account the discontinuities. The remaining structure is modelled by the FEM. The aims of this paper are to present a method with rotations coupling and to propose a way to reduce spurious wave reflections; it presents an application on a rock impact on a concrete slab.

1. Introduction

Particular attention must be paid during the design of certain civil engineering structures in order to predict their response under severe dynamic loading. One or several impacts of a projectile like an aircraft or a missile on a sensitive concrete structure may have disastrous consequences — for example, if the impact provides perforation on structure that has some protective functions (airtightness). This paper deals with a new numerical method to simulate an impact on a concrete structure. The structure is divided into two subdomains. On each of them, the best fit method is used. The approach uses a coupling between the Discrete Element (DE) Method and the Finite Element (FE) Method. In the vicinity of the impact, where important non-linear phenomena occur, the medium will be modelled by means of discrete elements. Far from this area, the response of the structure may be considered as linear elastic, and this complementary subdomain is modelled with the FE method.

Contrary to non-linear continuum methods [1], the DE method easily takes into account discontinuous phenomena. Our approach uses rigid sphere interactions, such as were used by Cundall [2]. The DE method is used in the impacted subdomain. These methods have been used first to model the behavior of granular materials, but also provide very accurate results for cohesive materials like concrete [3]. The studies of Camborde in 2D [4] or Rousseau et al. in 3D [5] have demonstrated the efficiency of such a discrete approach to deal with impact problems on reinforced concrete structures. They have also pointed out that such a method is limited to small structures because of the computation cost. The use of the FE method far from the impacted area is a way to reduce this limitation. Meshing softwares drastically reduce the time of modelling and the calculation may be faster than with a full DE approach because of the facility to handle different discretization sizes. Moreover the global behavior shows that the DE method is not needed on the whole structure. The aim of the coupled method is the prediction of both the local damage or the penetration of the projectile, and the global displacement of the structure.

Many studies deal with combined continuum/discrete methods, mainly with molecular dynamics and FE methods for the analysis of the fracture process at the atomic scale. A large review of such methods is proposed in Li and Liu [6], Rabczuk et al. [7]. Many applications have been carried out on concrete structures; in static by Azevedo et al. [8], Cusatis et al. [9] for the analysis of the fracturing process in heterogeneous materials; in transient analyses, and Onate and Rojek [10] studies the contact between FE and DE, Bicanic et al. [11] proposed a combined approach where the whole structure is modelled with FE which are disconnected and transformed to DE, depending on a stress criterion. Concerning the coupling between DE an FE, the works of Xiao and Belytschko [12] proposed a coupling using a bridging domain. Ben Dhia and Rateau [13] proposed the Arlequin method also based on a bridging domain with a weak formulation of the kinematic relations. In Xiao and Belytschko [12], the authors present some particular numerical simplification that improves the computational

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doi:10.1016/j.engstruct.2009.07.001

Nomenclature

$\overrightarrow{d_j}$	vector of the three displacements of the DE j
$\overrightarrow{\omega_j}$	vector of the three rotations of the DE j
$\overrightarrow{u_i}$	vector of the three displacement of FE node i
n_f	Number of FE nodes
n_d	Number of DE nodes
n_{fb}	Number of FE nodes in the bridging domain
n_{db}	Number of DE nodes in the bridging domain
n_b	Number of FE layers in the bridging domain
α_i	Bridging parameter for the FE node i
β_j	Bridging parameter for the DE j
r	Relaxation parameter
M_j	Mass of the DE j
J_j	Inertia of the DE j
m_i	Mass of the FE node i
$\overrightarrow{Fg_i}$	Generalized force vector on FE node i
$\overrightarrow{F_j^{tot}}$	Total force applied to DE j
$\overrightarrow{C_j^{tot}}$	Total couple applied to DE j
$\overrightarrow{\lambda^d}$	Lagrange multipliers related to displacement coupling
$\overrightarrow{\lambda^\omega}$	Lagrange multipliers related to rotation coupling
$\overline{\overline{k}}$	Displacement coupling matrix
$\overline{\overline{h}}$	Rotation coupling matrix

time and decreases the spurious wave reflection given by the interface.

The originality of our method is that we have to take into account the rotations of DE. Moreover, the numerical simplification discussed before does not strongly attenuate the high frequencies' reflection. We propose a new way to deal with the temporal DE boundary conditions. We introduce a temporal relaxation of the kinematic relations. This method strongly attenuates the spurious reflection due to different size discretization between the two methods. This method is equivalent to the use of the penalty method with a penalty parameter adapted to each degree of freedom (dof).

The discrete model and the main difficulties of coupling are first presented. Details of the coupling method are given, and a special emphasis on spurious wave reflections is carried out. Then we present our method to attenuate spurious reflections. It uses a relaxation of the Lagrange multipliers associated to the kinematic continuity. At the end, DE and DE/FE simulations of an impact on a concrete slab are compared.

2. Model description

The problem deals with fast transient dynamics in two subdomains of a concrete structure. One subdomain is modelled with DE, the other one with FE. This separation is done *a priori* from an estimation of the size of the damageable area. This step may be estimated from experimental analyses on a concrete slab [14] or from design rules (Eurocode 2, [15]). It depends on the kind of impact (soft [16], hard [17]), the geometry of the structure and, of course, the velocity of the impactor. Lots of experiments are accessible to estimate the size of the damageable part of the structure, so as to know the minimum size of the DE model. Moreover, we can check *a posteriori* that the stresses in FE domain stay in the elastic domain of the media. Another way is to verify that DE of the bridging domain have stayed in the elastic domain.

2.1. Discrete model

The domain, where discontinuities and non-linearities occur, is modelled with the DE method. Previous studies have demonstrated the efficiency of this approach to analyze structures under high deformation or many non linearities, in 2D [18] or 3D [19], in statics [9] or dynamics [20]. Our model is close to the one proposed by Cundall and Stack [2]. The heterogeneous medium is modelled by randomly positioned rigid spheres of different radii in interaction, link or contact. To represent the cohesive property of concrete, two elements can interact without being in contact. In [21], Hentz et al. presents a large description of the DE model. A modified Mohr–Coulomb criterion associated with softening is used to model the cohesive behavior of the material, and a classical friction constitutive behavior is used between the elements in contact. The material is modelled at a macro-scale, with the size of the DE being larger than the aggregate size. Nevertheless, the DE size will be as small as possible with respect to the computational time. Finally, an identification process is used to ensure that the model is predictive [21].

The DE degrees of freedom (dof) are three displacements and three rotations, so three more than the node of the FE model.

More details about complete model of the interaction laws and damage are available in Rousseau et al. [22] or Hentz et al. [21].

2.2. Continuum model

Far from the impacted area, the structure is modelled with the FE method under the small perturbation assumption. This assumption might be seen as a main constraint of the model, but the FE stresses can be used as an error indicator on the assumption, and it is still possible to enlarge the DE domain with, of course, a loss on the computation time or on the discretization size. Another indicator can be the damage of DE next to FE domain. In this paper, we focus on simulation with localized damage. The FE method is used to reduce the times of modelling (meshing) and computation by reducing the number of DE. The characteristic size of FE is a function of the structure size and its geometry. The FE size is much larger than the DE size (the ratio of discretization size is about 5).

2.3. Main difficulties of the coupling

The coupling can be realized by means of an edge to edge method or with a bridging domain [12] where energy is taken as a linear combination of each model by using a bridging parameter. In our case, due to the random positioning of DE, this model naturally takes into account the non regular boundaries. In some cases, particular treatment of the bridging domain is a way to reduce reflections that appear due to the model.

The numbers of dof in each model are not the same. In addition to the three displacements, the DE node also has three dof in rotation. The kinematic coupling of DE dof and FE dof must take into account the rotation continuity. Coupling rotations decreased error on discrete rotations, but had a slight influence on the displacements. For example, in a 3D case where the theoretical displacements and rotations were known, we obtained the following errors: 5% on discrete rotations and less than 0.5% on displacements. Numerical and physical experiments [23] show that, under the small perturbation assumption, the discrete rotation is the rigid rotation linked to the antisymmetric part of the displacement gradient. In the continuum model, under small strain, the displacement of N in the neighborhood of M is given by:

$$\vec{U}(N) \cong \vec{U}(M) + \overline{\overline{\varepsilon}}(M)\,\overrightarrow{MN} + \overline{\overline{\omega}}\,\overrightarrow{MN}$$
$$\text{with } \overline{\overline{\omega}} = \frac{1}{2}\left(\overline{\overline{grad}}\,\vec{U} - \overline{\overline{grad}}^{\mathrm{T}}\,\vec{U}\right). \quad (1)$$

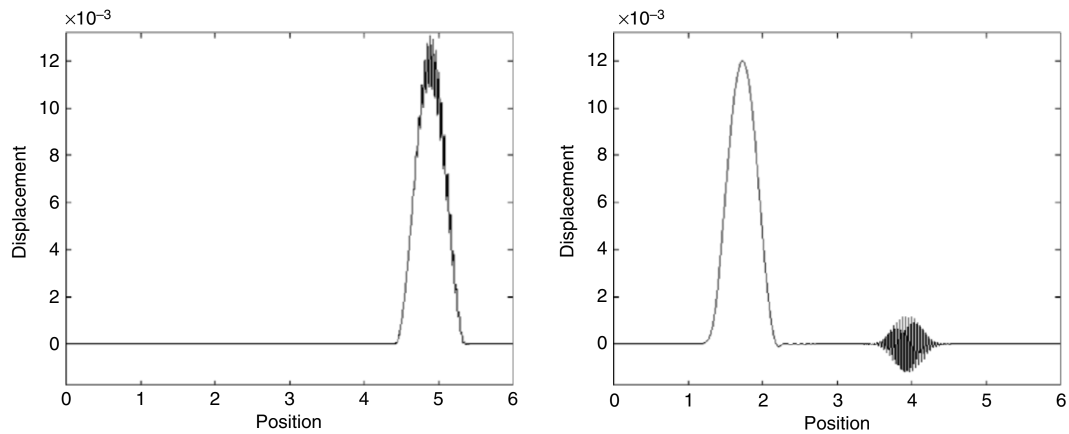

Fig. 1. Wave in DE model (left) and Spurious wave (right).

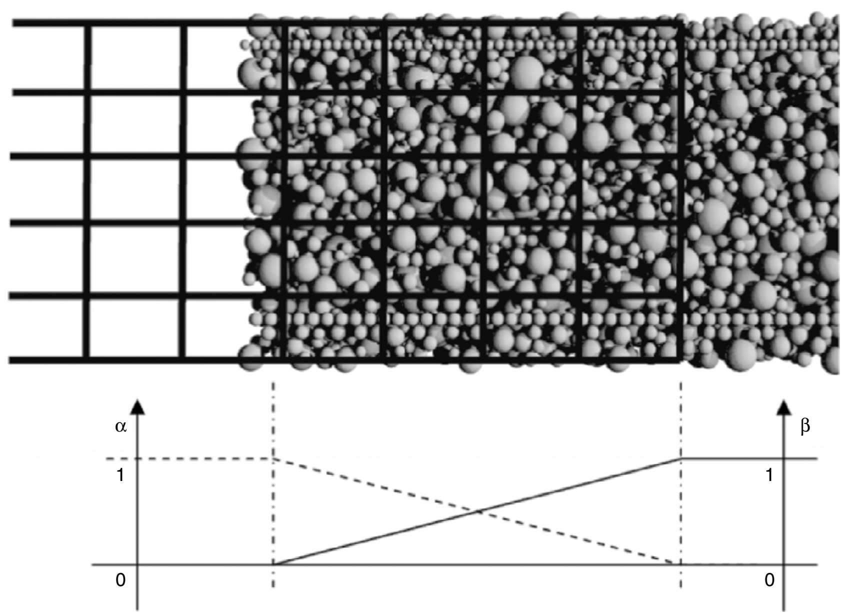

Fig. 2. Bridging domain and bridging parameter.

The displacement is written as the sum of a rigid body translation, one part due to strain and one due to rotation.

Another difficulty is that the discretization size is not uniform between the two models. The discretization size has a direct effect on the frequency range of the model. It is a key point in transient dynamics. A high frequency wave propagating from the DE model (fine) to the FE model (coarse) will introduce spurious wave reflections if the frequency is greater than the cut-off frequency of the FE model. The impact on reinforced concrete structure leads to a large frequency range. The experiments of Zineddin and Krauthammer [24] present high frequencies.

Fig. 1 shows an example of this reflection at the interface between the two models. It is a 1D model, where half of the structure (left) is modelled with coarse regular FE, and the other half with a fine DE model. The displacement wave is generated in displacement on the right hand boundary of the DE model by a sum of two low and high frequencies. The low frequency is transmitted, but the high frequency is fully reflected. This spurious reflection has to be suppressed in order to predict the correct response of the structure in the impacted area. It is to be noticed that such a problem also appears by replacing the fine DE model by a fine FE model. It is only due to the size of the coarse mesh that cannot represent short wave length.

3. The coupled method

3.1. Methods

The proposed method uses a bridging domain in which the Hamiltonian is taken as a linear combination of discrete and continuum Hamiltonians. The bridging parameters α and $1-\alpha$ are introduced respectively for FE and DE. They vary linearly, between 0 and 1, inside the bridging domain, and they are constant in the thickness of the structure. They are introduced to ensure the continuity of the energetic ration between DE model and FE model. The size of the bridging domain is defined by the parameter n_b corresponding to the number of FE layers in the bridging domain. Fig. 2 presents variations of bridging parameters for a bridging domain where $n_b = 4$. The Eq. (2) defines the expression of the Hamiltonian.

$$H = \alpha H_{FE} + (1-\alpha) H_{DE}. \tag{2}$$

In the bridging domain, the DE dof are linked to FE dof through the coupling relations, which can be written at the global scale by (3) and (4) or at the node scale by (5) and (6). Here, we separate translation $\vec{d}_b$ and rotation $\vec{\omega}_b$ dof of the discrete elements. Finally, there are as many coupling relations as DE dof in the bridging